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CONTENTS OF VOLUME 19

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MEASUREMENT OF LOW RESISTANCE BY MEANS OF THE WHEATSTONE BRIDGE

BY

FRANK WENNER, Physicist
ALVA SMITH, Associate Physicist
Bureau of Standards

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MEASUREMENT OF LOW RESISTANCE BY MEANS OF THE WHEATSTONE BRIDGE.

By Frank Wenner and Alva Smith.

ABSTRACT.

Resistances which are small can be definite only if they have distinct current and potential terminals.

In the procedure proposed the current terminals serve for connecting the resistance into the bridge circuit while the potential terminals serve as branch points. This puts the connecting resistances into arms adjacent to the arm containing the unknown resistance. Therefore their effect is made smaller than for the usual arrangement in the ratio of the unknown resistance to the resistance of the adjacent arms.

In cases where the errors still caused by the connecting resistances need be considered, two supplementary measurements with the same apparatus may be made practically to eliminate them.

With a recently calibrated bridge a resistance of the order of 0.001 ohm was measured to an accuracy of 0.6 per cent directly, and 0.03 per cent after applying the correction obtained by the supplementary measurement.

The values of the resistances are obtained from the data for the most part by addition and subtraction rather than by multiplication and division.

CONTENTS.

1. INTRODUCTION.

The engineer or the physicist often would like to know the resistance of a conductor having a resistance as low as 0.01 ohm or even 0.001 ohm. The unknown resistance may be that of a shunt, a reel of insulated cable, a sample for conductivity measurement or other low resistance whose value is required to an accuracy of a half of 1 per cent or better. Usually, however, a Thomson bridge or other special equipment is considered essential for such a measurement, but may not always be at hand. The purpose of this paper is to show that measurements of such conductors may be made to a fairly high accuracy by means of the Wheatstone bridge with the usual accessory apparatus.

2. NECESSITY FOR DEFINITE TERMINALS.

If a conductor of such low resistance is to have its resistance definite, it must have distinct current and potential terminals. Most shunts are provided with these. Other conductors to be tested should be supplied with such terminals. In case of wires and cables, if the ends are taken for two of the terminals each of the other terminals should be placed in from the ends a distance corresponding to at least several diameters of the conductor. The resistance measured then is of that part of the conductor which is between the inner terminals. A knife edge or rubbing contact should not be used as a terminal except under conditions such that there is little or no current through it, or its resistance does not materially affect the results obtained. Terminals should be soldered or screw connected in cases in which their resistance enters directly into measurements.

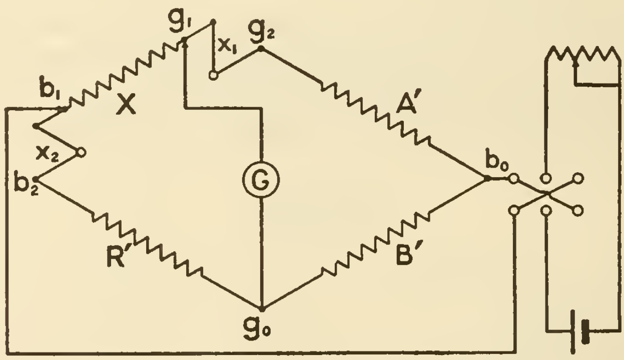

FIG. 1.—*Diagram showing low-resistance X connected into Wheatstone bridge so as to add the connecting resistances X_1 and X_2 to the adjacent arms.*

3. BRIDGE ARRANGEMENT.

The bridge to be discussed is represented in Figure 1. The resistance X to be measured is connected into the bridge by leads connecting the current terminals of the unknown to the X terminals of the bridge. Each of these connecting leads should have a low resistance, preferably not over 0.001 ohm. One of the battery leads is connected to the potential terminal b_1; one of the galvanometer terminals is connected to the other potential terminal g_1. The other battery lead is permanently connected to the junction b_0 between the ratio arms; the other galvanometer lead is permanently connected to the junction g_0 between the

higher ratio arm and the rheostat arm. The ratio arm A' adjacent to the unknown resistance is set at its lowest or the next to the lowest value and the other ratio arm B' set so that the rheostat arm R' reads, at balance, at least 500 times larger than the unknown resistance.

In the battery circuit there should be included a reversing switch, by means of which the current through the bridge may be opened or closed in either direction. A rheostat should be used in the battery circuit to limit the current to that which the lower ratio arm can carry without an excessive rise in temperature.

The characters used in Figure 1 to designate the arms of the bridge also represent the respective resistance of these arms. x_1 and x_2 are the resistance between the potential terminals of X, and A' and R', respectively.

4. PRIMARY MEASUREMENT.

When the resistances x_1 and x_2 are taken into consideration, the usual bridge relation gives

$$X = \frac{A' + x_1}{B'} (R' + x_2) \tag{1}$$

Obviously, A', B', and R' must each be known to as high a precision as is sought in the measurement of X. On the other hand x_1 and x_2 will be small in comparison, respectively, with A' and R' and, therefore, need not be known to such high precision. In fact, they may be neglected altogether where high accuracy is not required. As R' is usually small, it is necessary to read deflections of the galvanometer and determine R' by interpolation to less than 1/10 of the smallest change that can be made in the rhoestat arm.

If, as a first approximation, we ignore x_1 and x_2, equation (1) becomes

$$X = \frac{A' R'}{B'} \tag{2}$$

It is presumed that the bridge is in good adjustment or that corrections to the various readings are known and will be applied.

The following numerical examples taken from laboratory data will illustrate the accuracy that may be expected in the use of this equation. Appropriate corrections have been applied to the

readings of the bridge arms. The data given in these examples represent, therefore, actual resistances, not merely readings, of these arms. For a first example:

$$A' = 1.00045$$
$$B' = 1000.0$$
$$R' = 9.9898 \text{ ohms.}$$

These data give

$$X = 0.009994 \text{ ohm.}$$

The conductor under test in this case was a 0.01 ohm standard whose accepted value is 0.0100025 ohm. It follows, therefore, that the error in the measured value amounts to but 0.08 per cent. For a second example:

$$A' = 0.10023$$
$$B' = 1000.0$$
$$R' = 9.9188 \text{ ohms.}$$

These data give

$$X = 0.0009942 \text{ ohm.}$$

The conductor under test in this case was a 0.001 ohm standard whose accepted value is 0.000999997 ohm. The error in the measured value in this case is 0.58 per cent.

The errors arising from the above procedure are very small compared with those that would exist if the usual procedure with the Wheatstone bridge were followed in the measurement of these low resistances. These errors are low because the connecting resistances to the unknown have been thrown over into the adjacent arms of the bridge, which are high compared with the connecting resistances; whereas, with the usual arrangement, they would be included with the unknown, and as they may be of the same order of magnitude might introduce an error of the order of 100 per cent. For many purposes the accuracy of the above procedure is sufficient.

5. SECONDARY MEASUREMENT.

However, where higher accuracy is desired it can be obtained by taking into consideration the small effects of the connecting resistances x_1 and x_2, which were neglected in the preceding calculation. This requires two additional measurements, but these may be made without additional apparatus. Further, neither of these need to be made to a high accuracy, since they serve only for the purpose of determining small corrections to the

primary measurement. We shall, therefore, refer to these as secondary measurements.

One of these secondary measurements is made with the galvanometer lead transferred from g_1 to g_2, the other connections remaining as for the primary measurement. The bridge is then balanced by adjustment of the rheostat arm only. If R'_1 is the resistance of the rheostat arm, the relation between the resistances is

$$\frac{X+x_1}{R'_1+x_2}=\frac{A'}{B'} \tag{3}$$

The other secondary measurement is made with the galvanometer lead transferred from g_2 back to g_1 and the battery lead transferred from b_1 to b_2. The rheostat arm is again adjusted until balance is obtained. If R'_2 is the resistance of the rheostat arm the relation between the resistances is

$$\frac{X+x_2}{R'_2}=\frac{A'+x_1}{B'} \tag{4}$$

Equations (1), (3), and (4) are three independent relations between X, x_1, x_2, and the resistances of the bridge arms, from which it is possible to determine X, x_1, and x_2. However, values for x_1 and x_2 usually are not desired, and an exact solution of the three equations for X leads to a complicated expression from which the value can not readily be calculated. To obviate this difficulty, we shall write

$A+a$ for A'	$R+r_1$ for R'_1
$B+b$ for B'	$R+r_2$ for R'_2
$R+r$ for R'	

where A, B, R are simple nominal values, such as 0.1, 0.2, 0.5, 1, 2, 5, 10, etc., ohms, and a, b, r are the small amounts by which the actual resistance of the bridge arms exceed these simple nominal values.

Because of the approximations to be made later in the equations, A and B should not differ from A' and B' by as much as 1 per cent and R should not differ from R' by more than a few per cent, if an accuracy better than 0.1 per cent is desired. In some cases, therefore, it will not be possible to choose for R as simple a value as that indicated above, or as for A and B. However, in all cases the simplest possible values consistent with this limitation should be chosen for A, B, and R.

For example should

$$A' = 1.0022$$
$$B' = 1003.3$$
$$\text{and } R' = 25.354 \text{ ohms}$$

we would choose

$$A = 1$$
$$B = 1{,}000$$
$$\text{and } R = 25 \text{ ohms.}$$

If these changes in notation are made, equation (1) becomes

$$X = \frac{AR\left(1 + \frac{a}{A} + \frac{x_1}{A}\right)\left(1 + \frac{r}{R} + \frac{x_2}{R}\right)}{B\left(1 + \frac{b}{B}\right)} \tag{5}$$

Since the terms containing x_1 and x_2 are small compared with unity, they need not be known to a high precision. Therefore, in equation (3) we may neglect x_2, consider $\frac{A'}{B'} = \frac{A}{B}$, and $X = \frac{AR}{B}$. This gives

$$x_1 = \frac{A}{B}(r_1 - r) \tag{6}$$

Likewise, in equation (4) we may neglect x_1, consider $\frac{A'}{B'} = \frac{A}{B}$, and $X = \frac{AR}{B}$. This gives

$$x_2 = \frac{A}{B}(r_2 - r) \tag{7}$$

These values of x_1 and x_2 substituted in equation (5) give

$$X = \frac{AR}{B}\left[1 + \frac{a}{A} - \frac{b}{B} + \frac{r}{R} + \frac{r_1 - r}{B} + \frac{A\,(r_2 - r)}{BR} + \frac{r\,(r_1 - r)}{BR}\right] \tag{8}$$

Here we have neglected squares and cross products of small quantities, except one which may sometimes be significant.

It is obvious that an alternative procedure consists in computing the numerical values of x_1 and x_2 from equations (3) and (4), respectively. In this computation x_2 is neglected in equation (3) and x_1 in equation (4), and the value of X obtained from equation (2) is used. The computed values of x_1 and x_2 are then substituted back in equation (1), from which the corrected value of X can be obtained.

The examples discussed above will now be extended to show the effect of the two auxiliary measurements upon the accuracy attainable.

For the first example:

$$A' = 1.00045,\ B' = 1{,}000.00,\ R' = 9.9898$$
$$R_1' = 10.480$$
$$\text{and } R_2' = 10.72 \text{ ohms.}$$

If we take $A = 1$, $B = 1{,}000$, $R = 10$,
it follows that $a = 0.00045$, $b = 0.00$, $r = -0.0102$

$$r_1 = +0.48$$
$$\text{and } r_2 = +0.72 \text{ ohm.}$$

These values substituted in the right-hand member of equation (8) give

$$X = 0.01\ (1 + 0.00045 - 0.00102 + 0.00049 + 0.00007) \text{ ohm}$$
$$= 0.0099999 \text{ ohm.}$$

This differs from the known value by 0.025 per cent, whereas the value obtained by the primary measurement alone differs from the known value by 0.08 per cent.

For the second example:

$$A' = 0.10023,\ B' = 1{,}000.0,\qquad R' = 9.9188$$
$$R'_1 = 15.45$$
$$\text{and } R'_2 = 14.1 \text{ ohms.}$$

If we take $A = 0.1$, $B = 1{,}000.0$, $R = 10$ ohms, it follows that

$$a = 0.00023,\ b = 0.0,\ r = -\ .0812$$
$$r_1 = +5.45$$
$$\text{and } r_2 = +4.1 \text{ ohms.}$$

These values substituted in the right-hand member of equation (8) give

$$X = 0.001\ (1 + 0.0023 - 0.00812 + 0.00553 + 0.00004 - 0.00004) \text{ ohm}$$
$$= 0.0009997 \text{ ohm.}$$

This value differs from the known value by only 0.03 per cent, whereas the value obtained by the primary measurement alone differs from the known value by 0.58 per cent.

6. DISCUSSION OF RESULTS.

Inspection of these results shows that in measuring the resistance of 0.01 ohm, the corrections resulting from the secondary measurements reduced the error to about one-third of that existing in

the result obtained from the primary measurement alone. In measuring the resistance of 0.001 ohm, the error was reduced by the secondary measurements to about $\frac{1}{20}$ of that existing when only the primary measurement was used. This illustrates the importance of the secondary measurements when the resistance of the conductor under test is of the same order of magnitude or smaller than the lead and terminal resistances, x_1 and x_2.

In obtaining the data used in these examples, a recently calibrated Wolff bridge was used. In this particular type of bridge, resistance is added to the arms of the bridge by removal of a plug. The smallest step in the rheostat arm is 0.1 ohm, so it was necessary to interpolate to get the next two significant figures in the rheostat reading. The smallest setting of the rheostat arm is also 0.1 ohm. The apparatus was set up on one day and some preliminary tests made; on the following day a set of five readings was made on the primary measurement for each conductor under test, fewer secondary measurements being made in each case. The values used in our examples represent the means of each set of five primary measurements. The average deviation of the individual values from the mean was, for the 0.01 ohm resistance, 0.011 per cent and for the 0.001 ohm resistance, 0.005 per cent. The accuracy reported in these measurements is not to be expected except by an experienced observer using a standardized bridge in which the ratio arms may be set at least as low as 1 ohm and the rheostat arm is adjustable in steps as small as 0.1 ohm. Neither should one expect an accuracy such as is attainable with the Thomson-bridge method. The procedure, however, is but little more difficult than that required in the usual measurements of resistance in which an effort is made to obtain a corresponding accuracy. We present it, therefore, for use where a fairly high accuracy is desired in the measurement of a low resistance and where more suitable apparatus is not available.

The general procedure followed for eliminating errors on account of the resistance of leads and terminals is not new; it has been in use in the Bureau of Standards, the National Physical Laboratory, and possibly elsewhere, for years. However, in so far as we are aware, its application to the measurement of low resistances by means of the usual type of Wheatstone bridge has not been considered previously. Neither has it been generally appreciated that for such a case only one of the three measurements required need be made to a high precision, nor that a fairly high accuracy may be obtained by a single measurement.

7. SUMMARY.

1. It is pointed out again that if a low resistance is to be definite, it must have distinct current and potential terminals.

2. By using the potential terminals of the unknown as branch points of the bridge, the terminal and connecting resistances to the unknown are thrown into the adjacent arms of the bridge. By making the resistances of these arms high compared with the connecting resistances, the error caused by the connecting resistances is made at once relatively small.

3. In cases where it is desirable to corrcet for the small errors mentioned above, it is shown that this may be done by two additional or auxiliary measurements with the same apparatus.

4. In the measurement of a resistance of the order of 0.001 ohm an accuracy of 0.6 per cent was obtained by direct measurement and 0.03 per cent when corrections obtained by the auxiliary measurements were applied.

WASHINGTON, July 24, 1923.

INDEX TO VOLUME 19